

Limits on the Time Evolution of Space Dimensions from Newton's Constant

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Abstract

Limits are imposed upon the possible rate of change of extra spatial dimensions in a decrumpling model Universe with time variable spatial dimensions (TVSD) by considering the time variation of (1+3)-dimensional Newton's constant. Previous studies on the time variation of (1+3)-dimensional Newton's constant in TVSD theory had not been included the effects of the volume of the extra dimensions and the effects of the surface area of the unit sphere in D-space dimensions. Our main result is that the absolute value of the present rate of change of spatial dimensions to be less than about 10^{-14}yr^{-1} . Our results would appear to provide a *prima facie* case for ruling the TVSD model out. We show that based on observational bounds on the present-day variation of Newton's constant, one would have to conclude that the spatial dimension of the Universe when the Universe was "at the Planck scale" to be less than or equal to 3.09. If the dimension of space when the Universe was "at the Planck scale" is constrained to be fractional and very close to 3, then the whole edifice of TVSD model loses credibility.

1 Introduction

Although time variability of spatial dimensions have not been firmly achieved in experiments and theories, such dynamical behavior of the spatial dimensions should not be ruled out in the context of cosmology and astroparticle physics.

This article studies the time variation of the (1+3)-dimensional Newton's constant in a model Universe with time variable space dimension (TVSD) following the original idea presented in "A model Universe with variable dimension: expansion as decrumpling" [1].

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Decrumpling model of the Universe is a new sort of cosmological scenario based on the assumption that the basic building blocks of the spacetime are fractally structured [1]-[7]. In the original papers [1], the spatial dimension of the Universe was considered as a continuous time dependent variable. As the Universe expands, its spatial dimension decreases continuously, thereby generating what has been named a decrumpling Universe. Then this model has been overlooked and the quantum cosmological aspects, as well as, a possible test theory for studying time evolution of Newton's constant have also been discussed [2, 3]. Chaotic inflation in TVSD theory and its dynamical solutions have also been studied in Refs. [4, 5].

The idea of changing the spatial dimension of the Universe dynamically as has been suggested in the original papers [1] is a bold one and perhaps seems to be unique and novel but we believe that daring and speculative ideas like this should be explored.

There have been attempts to identify a fractal dimension for the matter distribution in space using either cosmic microwave background radiation (CMBR) or galaxy distribution [8, 9]. Aside from the actual dimension of space or the matter distribution in it, it is interesting to study the cosmological consequences of a fractal and variable space dimension. All critiques of space dimensionality other than 3 rely upon cosmologically small scale observations [10]. Therefore, one could ask about the consequences of a dynamical space dimension in cosmological time and space scales. A proposed way of handling such a concept is using the idea of decrumpling coming from polymer physics [11, 12, 13]. In recent years, the physics community has witnessed a spectacular revival in interest for the evolution of extra spatial dimensions [14, 15, 16, 17, 18, 19, 20, 21, 22]. The topic of the fractal dimension has also been studied in Refs. [23, 24, 25, 26, 27, 28, 29, 30]. The evolution of the fractal dimension of a self-similar Universe in the context of Newton's gravitation has been discussed in Ref. [31].

Here, we will be concerned with the approaches proposed in Ref. [1] where the cosmic expansion of the Universe is named decrumpling expansion and is due to the decrease of the spatial dimensions. The most important difference between TVSD model/theory and other attempts about the evolution of the spatial dimension is that in this theory the number of the extra spatial dimensions changes with time while in other theories the size of the extra spatial dimension is a dynamical parameter, see Refs. [14, 15].

Another subject which lately has attracted much attention is the time

variation of the physical constants for example the fine structure constant and the Newton's gravitational constant (see Ref. [32] for a thorough review or Refs. [34, 35] for a brief study).

In the previous studies Refs. [3, 36] about the effective time variation of Newton's constant in TVSD model the effects of the surface area of the unit sphere in D -space dimensions had not been included and the effects of the volume of the extra spatial dimensions had been included for closed Universe which is wrong because we know based on recent observational data that our Universe is flat. Here, our study will include correctly these important effects and will give a lower bound about 10^{-14}yr^{-1} on the rate of the change of the spatial dimension based on the time variation of $(1+3)$ -dimensional Newton's constant. We use the relationship between Newton's constant in $(1+3)$ and $(1+D)$ -dimensional theories as obtained by Gauss' law in Ref. [33].

In this paper we will answer to this main question can such models be ruled out observationally? Our results would appear to do so. In particular, our conclusion is that based on observational bounds on the present-day variation of Newton's constant, one would have to conclude that the space dimension when the Universe was "at the Planck scale" to be less than or equal to 3.09. If the dimension of space when the Universe was "at the Planck scale" is constrained to be fractional and very close to 3, then the whole edifice of this model loses credibility.

We will use a natural unit system that sets k_B , c and \hbar all equal to 1, so that $\ell_P = M_P^{-1} = \sqrt{G}$. To read easily this article we also use the notation D_t instead of $D(t)$ that means the space dimension D is as a function of cosmic time.

The plan of this article is as follows. Since TVSD model/theory is so far from the mainstream of current researches, in section 2 we will give a brief review of the idea of TVSD theory and of its physical content presented in Refs. [1]-[7]. In section 3, we will confront this idea with the time variation of Newton's constant, showing that the effect of the total volume of the extra spatial dimension and the surface area of the unit sphere in D -space dimension are important while in previous studies in Ref. [3] have not been included. In section 4, we explain that our results would appear to provide a *prima facie* case for ruling the TVSD model out and we also study the time variation of the spatial dimension from viewpoint of the anthropic cosmological principle. Finally, we discuss our results and conclude in section

5.

2 Review of TVSD Theory

2.1 Motivation for choosing a Universe with TVSD

Problems of the standard model and its difficulties with the concept of quantum gravity and the early Universe at the Planck time provide us enough reasons to look for viable model Universes. Moreover, the ongoing experiments related to CMB will provide us a wealth of data suitable to test all the theories of spacetime and gravity. Even the act of verifying cosmological models based on general relativity needs looking for viable theories differing from it to see the degree of its testability and viability. These are the main reasons we are studying decrumpling Universe based on time variable space dimensions. It has been shown in Ref. [2] that this idea can be implemented successfully in a gravitational theory and cosmological model based on it. The free parameter of the theory may then be fixed by observational data [3].

The idea of having spacetime dimensions other than $1 + 3$ goes back to Kaluza-Klein theory. The generalization of this concept to string theories with space dimension more than three, but still an integer, and a constant is well known [37, 38]. This, being considered for the high energy limit in the Universe or for the dimension of space at the Planck time, has encouraged people to suggest that the dimension of space in the lower energy limit, or for the actual structured Universe, be other than three.

A possible time dependence of spacetime dimensionality is derived as an effect of entropy conservation in Ref. [22]. There it turns out that, going back in time, the dimension increases first very slowly up to about the Planck time, and increases more rapidly thereafter. This generic trend is insensitive to the assigned entropy value. In fact a minimum value for the size of the Universe, being about the Planckian size, is also obtained. The dynamical model we are going to follow has the same time behavior for the space dimension. A minimum size for the Universe is also built into our dynamical model.

Our treatment in this paper is based on a cosmological model, where the number of spatial dimensions decreases continuously as the Universe expands, presented in the pioneer works [1]. A proposed way of handling such

a concept is using the idea of decrumpling coming from polymer physics [11, 12, 13]. In this model the fundamental building blocks of the Universe are like cells being arbitrary dimensions and having, in each dimension, a characteristic size δ which maybe of the order of the Planck length $\mathcal{O}(10^{-33}$ cm) or even smaller. These “space cells” are embedded in a \mathcal{D} space, where \mathcal{D} may be up to infinity. Therefore, the space dimension of the Universe depends on how these fundamental cells are configured in this embedding space. The Universe may have begun from a very crumpled state having a very high dimension \mathcal{D} and a size δ , then have lost dimension through a uniform decrumpling which we see like a uniform expansion. The expansion of space, being now understood like a decrumpling of cosmic space, reduces the spacetime dimension continuously from $\mathcal{D} + 1$ to the present value $D_0 + 1$. In this picture, the Universe can have any space dimension. As it expands, the number of spatial dimensions decreases continuously. The physical process that causes or necessitates such a decrease in the number of spatial dimensions comes from how these fundamental cells are embedded in a \mathcal{D} space.

As an example, take a limited number of small three-dimensional beads. Depending on how these beads are embedded in space they can configure to a one-dimensional string, two-dimensional sheet, or three-dimensional sphere. This is the picture we are familiar with from the concept of crumpling in polymer physics where a crumpled polymer has a dimension more than 1. Or take the picture of a clay which can be like a three-dimensional sphere, or a two-dimensional sheet, or even a one-dimensional string, a picture based on the theory of fluid membranes.

The major formal difficulty to implement this idea in a spacetime theory with variable space dimension is that the measure of the integral of the action is variable and therefore some part of integrand. However, taking into account the cosmological principle, i.e. the homogeneity and isotropy of space, the formulations are simplified substantially. It then becomes possible to formulate a Lagrangian for the theory and write down the corresponding field equations. This Lagrangian is however not unique [2].

The original decrumpling model of the Universe seems to be singularity free, having two turning points for the space dimension [1]. The authors of Ref. [6] criticize the way of generalizing the standard cosmological model to arbitrary variable space dimension used in Ref. [1] and propose another way of writing the field equations. Their model shows no upper bound for the dimension of space, see also Ref. [7].

Later on this scenario was extended to the class of multidimensional cosmological models, where extra factor spaces play the role of the matter fields. In this multidimensional cosmological model an inflationary solution was found together with the prediction that the Universe starts from a nonsingular spacetime [17]. Moreover, the dynamics of the dimensions in factor space cosmology has been studied in Ref. [17].

A new way to generalize the gravitational action in constant dimension to the case of dynamical dimension is proposed in Ref. [2]. There, it is shown that the generalization of the gravitational action to the dynamical dimension is not unique. Moreover, in contrast to the earlier works in Ref. [1], the dependence of the measure of the action on space dimension is taken into account. This new decrumpling model is studied in detail in Ref. [2]. The generalization of the action, the Lagrangian, the equations of motion to dynamical space dimension, the time evolution of the spatial dimension, numerical results for the turning points of the model, and its quantum cosmology within the concept of the Wheeler-DeWitt equation are derived. It is shown that the corresponding potential of the model has completely different behavior from the potential of the de Sitter minisuperspace in three-space. Imposing the appropriate boundary condition in the limit $a \rightarrow +\infty$, and using the semiclassical approximation, the wave function of the model is also obtained. It is then seen that in the limit of constant space dimension, the wave function is not well-defined. It can approach to the Hartle-Hawking wave function or to the modified Linde wave function, but not to that of Vilenkin. In the limit of constant spatial dimension, the probability density approaches to Vilenkin, Linde and others' proposal; i.e. to the probability density $\mathcal{P} \propto \exp(2S_E)$, or more generally $\exp(-2|S_E|)$, where S_E is the Euclidean action of the classical instanton solution. Chaotic inflation in TVSD scenario has been studied in Ref. [4] and its solutions are presented in Ref. [5]. In Ref. [3] the effective time variation of Newton's constant in TVSD theory has also been investigated.

2.2 Relation between the effective space dimension $D(t)$ and characteristic size of the Universe $a(t)$

Assume the Universe consists of a fixed number N of universal cells having a characteristic length δ in each of their dimensions. The volume of the

Universe at the time t depends on the configuration of the cells. It is easily seen that [1]

$$\text{vol}_D(\text{cell}) = \text{vol}_{D_0}(\text{cell})\delta^{D-D_0}. \quad (1)$$

Interpreting the radius of the Universe, a , as the radius of gyration of a crumpled “universal surface” [13], the volume of space can be written [1]

$$\begin{aligned} a^D &= N \text{vol}_D(\text{cell}) \\ &= N \text{vol}_{D_0}(\text{cell})\delta^{D-D_0} \\ &= a_0^{D_0} \delta^{D-D_0} \end{aligned} \quad (2)$$

or

$$\left(\frac{a}{\delta}\right)^D = \left(\frac{a_0}{\delta}\right)^{D_0} = e^C, \quad (3)$$

where C is a universal positive constant. Its value has a strong influence on the dynamics of spacetime, for example on the dimension of space, say, at the Planck time. Hence, it has physical and cosmological consequences and may be determined by observations. The zero subscript in any quantity, e.g. in a_0 and D_0 , denotes its present values. We coin the above relation as a “dimensional constraint” which relates the “scale factor” of our model Universe to the space dimension. In our formulation, we consider the comoving length of the Hubble radius at present time to be equal to one. So the interpretation of the scale factor as a physical length is valid. The dimensional constraint can be written in this form

$$\frac{1}{D} = \frac{1}{C} \ln \left(\frac{a}{a_0} \right) + \frac{1}{D_0}. \quad (4)$$

It is seen that by expansion of the Universe, the space dimension decreases. Note that in Eqs.(3) and (4), the space dimension is a function of cosmic time t . Time derivative of Eqs.(3) or (4) leads to

$$\dot{D} = -\frac{D^2 \dot{a}}{C a}. \quad (5)$$

It can be easily shown that the case of constant space dimension corresponds to when C tends to infinity. In other words, C depends on the number of fundamental cells. For $C \rightarrow +\infty$, the number of cells tends to infinity and $\delta \rightarrow 0$. In this limit, the dependence between the space dimensions and the radius of the Universe is removed, and consequently we have a constant space dimension.

Table 1: Values of C and δ for some values of D_P [2]. Time variation of space dimension today has also been calculated in terms of sec^{-1} and yr^{-1} .

| D_P | C | δ (cm) | $\dot{D} _0$ (sec^{-1}) | $\dot{D} _0$ (yr^{-1}) |
|-----------|-----------|---------------------------|------------------------------------|-----------------------------------|
| 3 | $+\infty$ | 0 | 0 | 0 |
| 4 | 1678.797 | 8.6158×10^{-216} | $-1.7374 \times 10^{-20} h_0$ | $-5.4827 \times 10^{-13} h_0$ |
| 10 | 599.571 | 1.4771×10^{-59} | $-4.8648 \times 10^{-20} h_0$ | $-1.5352 \times 10^{-12} h_0$ |
| 25 | 476.931 | 8.3810×10^{-42} | $-6.1158 \times 10^{-20} h_0$ | $-1.9299 \times 10^{-12} h_0$ |
| $+\infty$ | 419.699 | ℓ_P | $-6.9498 \times 10^{-20} h_0$ | $-2.1931 \times 10^{-12} h_0$ |

2.3 Physical Meaning of D_P

We define D_P as the space dimension of the Universe when the scale factor is equal to the Planck length ℓ_P . Taking $D_0 = 3$ and the scale of the Universe today to be the present value of the Hubble radius H_0^{-1} and the space dimension at the Planck length to be 4, 10, or 25, from Kaluza-Klein and superstring theories, we can obtain from Eqs. (3) and (4) the corresponding value of C and δ :

$$\frac{1}{D_P} = \frac{1}{C} \ln \left(\frac{\ell_P}{a_0} \right) + \frac{1}{D_0} = \frac{1}{C} \ln \left(\frac{\ell_P}{H_0^{-1}} \right) + \frac{1}{3}, \quad (6)$$

$$\delta = a_0 e^{-C/D_0} = H_0^{-1} e^{-C/3}. \quad (7)$$

In Table 1, values of C , δ and also $\dot{D}|_0$ for some interesting values of D_P are given². These values are calculated by assuming $D_0 = 3$ and $H_0^{-1} = 3000 h_0^{-1} \text{Mpc} = 9.2503 \times 10^{27} h_0^{-1} \text{cm}$, where $h_0 = 0.68 \pm 0.15$. Since the value of C and δ are not very sensitive to h_0 we take $h_0 = 1$.

2.4 Lagrangian formulations of the model and field equations

Usually, we are accustomed to work with an integer number of dimension, and therefore a non-integer total number of spatial dimensions looks peculiar. It is clear that for non-integer value of space dimensions, one cannot

²Our solar year (the time required for Earth to travel once around the Sun) is 365.24219 days, <http://www.mystro.com/leap.htm>.

define the metric tensor. To overcome this problem, we use a gravitational theory based on Lagrangian formulations. In Ref. [2], some shortcomings of the original Lagrangian formulation of the model proposed in [1] have been shown, regarding the fields equations and their results.

Let us define the action of the model for the special Friedmann-Robertson-Walker (FRW) metric in an arbitrary fixed space dimension D , and then try to generalize it to variable dimension. Now, take the metric in constant $D+1$ dimensions in the following form

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Sigma_k^2, \quad (8)$$

where $N(t)$ denotes the lapse function and $d\Sigma_k^2$ is the line element for a D -manifold of constant curvature $k = +1, 0, -1$. The Ricci scalar is given by

$$R = \frac{D}{N^2} \left\{ \frac{2\ddot{a}}{a} + (D-1) \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{N^2 k}{a^2} \right] - \frac{2\dot{a}\dot{N}}{aN} \right\}. \quad (9)$$

Substituting from Eq.(9) in the Einstein-Hilbert action for pure gravity,

$$S_G = \frac{1}{2\kappa} \int d^{(1+D)}x \sqrt{-g} R, \quad (10)$$

and using the Hawking-Ellis action of a perfect fluid for the model Universe with variable space dimension the following Lagrangian has been obtained [2]

$$L_I := -\frac{V_D}{2\kappa N} \left(\frac{a}{a_0} \right)^D D(D-1) \left[\left(\frac{\dot{a}}{a} \right)^2 - \frac{N^2 k}{a^2} \right] - \rho N V_D \left(\frac{a}{a_0} \right)^D, \quad (11)$$

where $\kappa = 8\pi M_P^{-2} = 8\pi G$, ρ the energy density, and V_D the volume of the space-like sections

$$V_D = \frac{2\pi^{(D+1)/2}}{\Gamma[(D+1)/2]}, \quad \text{closed Universe, } k = +1, \quad (12)$$

$$V_D = \frac{\pi^{(D/2)}}{\Gamma(D/2+1)} \chi_c^D, \quad \text{flat Universe, } k = 0, \quad (13)$$

$$V_D = \frac{2\pi^{(D/2)}}{\Gamma(D/2)} f(\chi_c), \quad \text{open Universe, } k = -1. \quad (14)$$

Here χ_C is a cut-off and $f(\chi_c)$ is a function thereof (see Ref. [2]).

Two appropriate questions are raised here. The first question is that the cut off in Eqs. (13) and (14) seems *ad hoc*, what determines this scale χ_c ? and the second question is that is the boundary term in the action of Eq. (10), more involved with arbitrary dimension? The answer to the first question is that in cosmology the proper or physical length are obtained from the comoving length by multiplication of the Friedmann scale factor $\ell_{\text{physical}} = a(t)\ell_{\text{comoving}}$. While the comoving length does not change with time, the proper length changes with time because of $a(t)$. We take the scale factor having the dimension of length and the comoving length is a dimensionless quantity. The comoving length is measured by a set of constant rulers, while the proper length is measured by a set of expanding or contracting rulers. The flat model, as we know it is correct for our Universe, is unbounded with infinite volume, and with infinite radius. So as explained in Appendix A of Ref. [2] for flat and open Universe the volume of the space-like sections are infinite and χ_c is used as a very large number so that we have

$$V_D \propto \int_0^{\chi_c} \chi^{D-1} d\chi,$$

for flat Universe with $k = 0$ and

$$V_D \propto \int_0^{\chi_c} \sinh^{D-1} \chi d\chi,$$

for open Universe with $k = -1$. The answer to the second question about the boundary term in the action of Eq. (10) is that in the original and substantial paper of [2] about this unorthodox model we did not considered this boundary term since by our knowledge this term does not have crucial effects on the dynamics of the model. For this reason we ignored the boundary term in the action of Eq. (10) in our original paper of [2]. One another answer to the second question is that based on the gravitational action as given in Weinberg's book [41] we can ignore the boundary term in the action of Eq. (10).

In the limit of constant space dimensions, or $D = D_0$, L_I approaches to the Einstein-Hilbert Lagrangian which is

$$L_I^0 := -\frac{V_{D_0}}{2\kappa_0 N} \left(\frac{a}{a_0}\right)^{D_0} D_0(D_0 - 1) \left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{N^2 k}{a^2} \right] - \rho N V_{D_0} \left(\frac{a}{a_0}\right)^{D_0}, \quad (15)$$

where $\kappa_0 = 8\pi G_0$ and the zero subscript in G_0 denotes its present value. So, Lagrangian L_I cannot abandon Einstein's gravity. Varying the Lagrangian L_I with respect to N and a , we find the following equations of motion in the gauge $N = 1$, respectively

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{2\kappa\rho}{D(D-1)}, \quad (16)$$

$$(D-1)\left\{\frac{\ddot{a}}{a} + \left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\left(-\frac{D^2}{2C}\frac{d\ln V_D}{dD} - 1 - \frac{D(2D-1)}{2C(D-1)} + \frac{D^2}{2D_0}\right)\right\} + \kappa p\left(-\frac{d\ln V_D}{dD}\frac{D}{C} - \frac{D}{C}\ln\frac{a}{a_0} + 1\right) = 0. \quad (17)$$

Using (5) and (16), the evolution equation of the space dimension can be obtained by

$$\dot{D}^2 = \frac{D^4}{C^2} \left[\frac{2\kappa\rho}{D(D-1)} - k\delta^{-2}e^{-2C/D} \right]. \quad (18)$$

The continuity equation of the model Universe with variable space dimension can be obtained by (16) and (17)

$$\frac{d}{dt} \left[\rho \left(\frac{a}{a_0} \right)^D V_D \right] + p \frac{d}{dt} \left[\left(\frac{a}{a_0} \right)^D V_D \right] = 0. \quad (19)$$

2.5 What was considered erroneously in Ref. [3]?

As we will show in the next section, there are the dramatic differences between the conclusions of this paper and our earlier paper (Ref. [3]) which also considered time variation of Newton's constant. Let us now describe a more detailed statement of precisely what was wrong with Ref. [3]?

In [3], we generalized a formulation of a one parameter test theory - in which β is a constant parameter - for the time variation of Newton's constant to the cases where β is not constant but a function of time

$$G = G_0 \left(\frac{t}{t_0} \right)^{\beta(t)}. \quad (20)$$

The time derivative of this equation yields

$$\frac{\dot{G}}{G} = \frac{\beta(t)}{t} + \dot{\beta}(t) \ln \left(\frac{t}{t_0} \right) \simeq \frac{\beta(t)}{t}, \quad (21)$$

Table 2: Values of D_P , C and β_0 , see Ref.[3].

| D_P | C | β_0 |
|-----------|-----------|-----------|
| 3 | $+\infty$ | 0 |
| 4 | 1678.797 | -0.004 |
| 10 | 599.571 | -0.012 |
| 25 | 476.931 | -0.015 |
| $+\infty$ | 419.699 | -0.017 |

where $t_0 \simeq 10^{17}$ sec is the present time and G_0 is the present value of Newton's constant. It is worth mentioning that $\beta(t)$ and its time derivative $\dot{\beta}(t)$ satisfy the following condition

$$\left| \frac{\dot{\beta}(t)}{\beta(t)} \right| \ll \left| \frac{1}{t \ln \frac{t}{t_0}} \right|. \quad (22)$$

This condition may not always be valid. Therefore it must be checked for each case. Data from big bang nucleosynthesis yields for the present value of constant β -parameter

$$|\beta_0| \leq 0.01. \quad (23)$$

Comparing the coefficients of $(\dot{a}/a)^2$ in L_I and L_I^0 , as given in Eq.(11) and (15) respectively, we obtain

$$\frac{G}{V_D D(D-1)} = \frac{G_0}{V_{D_0} D_0(D_0-1)}. \quad (24)$$

Time derivative of this equation with assuming a closed Friedmann Universe with $k = +1$ yields the present value of β for different values of D_P and C , see Table 2. Comparing the values of β_0 from Table 2 with observational data, as given by (23), one can rule TVSD models out with $D_P \geq 10$.

There are some criticisms to above treatments as given in [3]. Firstly why we considered a closed Universe while based on recent observational data we know our real Universe is flat. Secondly our approach in [3] is the comparison of the coefficient of $(\dot{a}/a)^2$ in Lagrangians L_I and L_I^0 . This comparison does not have reasonable credibility. In other words, in [3] about the effective time variation of Newton's constant in TVSD model the effects of the surface area of the unit sphere in D -space dimensions had not been included and the

effects of the volume of the extra spatial dimensions had been included for closed Universe which is wrong because we know by recent observational data that our Universe is flat. In this paper our treatment for the time variation of Newton's constant is based on a credible approach as explained in the next section.

3 Time Variation of Newton's Constant in TVSD Theory

3.1 Newton's constant in constant D -space dimension

Taking the space dimension is constant and has an arbitrary value, here we derive the exact relationship between Newton's gravitational constants $G_{(1+D)}$ and $G_{(1+3)}$, in $(1 + D)$ and in $(1 + 3)$ dimensional theories.

Using the force laws in $(1 + D)$ and $(1 + 3)$ dimensional theories, which are defined by

$$F_{(1+D)}(r) = G_{(1+D)} \frac{m_1 m_2}{r^{D-1}}, \quad (25)$$

$$F_{(1+3)}(r) = G_{(1+3)} \frac{m_1 m_2}{r^2}, \quad (26)$$

and the $(1 + D)$ dimensional Gauss' law, one can derive the exact relationship between the gravitational constants $G_{(1+D)}$, $G_{(1+3)}$ of the full $(1 + D)$ and compactified $(1 + 3)$ dimensional theories (see Ref. [33] for a more detailed explanation)

$$G_{(1+3)} = \frac{S_D}{4\pi} \frac{G_{(1+D)}}{V_{(D-3)}}, \quad (27)$$

where $S_D = 2\pi^{D/2}/\Gamma(D/2)$ is the surface area of the unit sphere in D spatial dimensions and $V_{(D-3)}$ is the volume of $(D - 3)$ extra spatial dimensions.

According to Eq. (27), in a Universe with constant D -space Newton's constant does not have any time variation because all quantities in Eq. (27) are constant.

3.2 Time variation of the effective $G_{(1+3)}$ from time variable space dimension

Let us now generalize Eq. (27) from constant D -space dimension to time-varying $D(t)$ -space dimension. We use the following substitution

$$D = \text{constant} \rightarrow D(t) := D_t,$$

where the t subscript in D_t means that D to be as a function of time. In previous references [1]-[7] the notation of D_t did not use for $D(t)$. Here the author use this notation to make more clear for the readers.

Therefore, the force law in TVSD theory reads

$$F_{(1+D_t)}(r) = G_{(1+D_t)} \frac{m_1 m_2}{r^{D_t-1}}, \quad (28)$$

and the exact relationship between Newton's constants in TVSD theory is

$$G_{(1+3)} = \frac{S_{D_t}}{4\pi} \frac{G_{(1+D_t)}}{V_{(D_t-3)}}, \quad (29)$$

where S_{D_t} is the surface area of the unit sphere in D_t spatial dimensions

$$S_{D_t} = \frac{2\pi^{D_t/2}}{\Gamma(D_t/2)}, \quad (30)$$

and $V_{(D_t-3)}$ is the volume of (D_t-3) extra spatial dimensions in TVSD theory

$$V_{(D_t-3)} \approx a^{D_t-3}, \quad (31)$$

where a is the scale factor, see the metric in Eq. (8).

It is easy to see that $G_{(1+D_t)}$, has variable dimension $[\text{length}]^{D_t-1}$ and $G_{(1+3)}$ has constant dimension $[\text{length}]^2$. Each time we have varying constants, it is very important to talk about dimensionless constant such as the fine structure constant (see Ref. [32]). In the case of the Newton's constant, what is meant is usually $G_{(1+3)}m^2$ (more exactly $G_{(1+3)}m^2/\hbar c$) where m is some mass. So, we introduce a fixed mass scale M that can be taken to be the Planck mass or some other fixed mass scale.

We take the fixed mass scale to be the Planck mass. Therefore, Eq. (29) takes the form

$$G_{(1+3)} = \frac{S_{D_t}}{4\pi} \frac{G_{(1+D_t)} M_P^{D_t-3}}{(M_P a)^{D_t-3}}. \quad (32)$$

The quantity $G_{(1+D_t)} M_P^{D_t-3}$ has dimension $[\text{length}]^2$ and we define it

$$\bar{G}_{(1+D_t)} \equiv G_{(1+D_t)} M_P^{D_t-3}. \quad (33)$$

It is worth mentioning that if we did not introduce a fixed mass scale the time variation of Eq. (29) runs into serious mathematical difficulties e.g. $\ln(a)$ which is meaningless because the scale factor a has dimension $[\text{length}]$.

One general feature of extra-dimensional theories, such as Kaluza-Klein and string theories, is that the “true” constants of nature are defined in the full higher dimensional theory so that the effective 4-dimensional constants depends, among other things, on the structure and size of the extra-dimensions. Any evolution of these sizes either in time or space, would lead to a spacetime dependence of the effective 4-dimensional constants, see Ref. [32]. So $\bar{G}_{(1+D_t)}$ is a “true” constant and we have $\dot{\bar{G}}_{(1+D_t)} = 0$.

Time derivative of Eq. (32) takes the form

$$\frac{\dot{G}_{(1+3)}}{G_{(1+3)}} = \frac{\dot{S}_{D_t}}{S_{D_t}} + \frac{\frac{d}{dt}[(M_P a)^{D_t-3}]}{[(M_P a)^{D_t-3}]}. \quad (34)$$

From Eqs. (3) and (5), we have

$$a = \delta e^{C/D_t}, \quad (35)$$

$$\delta = a_0 e^{-C/D_0}. \quad (36)$$

Using these equations and Eq. (30), we get

$$\frac{\dot{G}_{(1+3)}}{G_{(1+3)}} = \dot{D}_t \left[\frac{1}{2} \ln \pi - \frac{1}{2} \psi(D_t/2) - \frac{C}{D_P} + \frac{C}{D_t} \right] + (D_t - 3) \frac{\dot{a}}{a}, \quad (37)$$

where Euler’s psi function ψ is the logarithmic derivative of the gamma function $\psi(x) \equiv \Gamma'(x)/\Gamma(x)$. Finally, using Eq. (5) we can rewrite Eq. (37) in the form

$$\frac{\dot{G}_{(1+3)}}{G_{(1+3)}} = -\frac{D_t^2}{C} \frac{\dot{a}}{a} \left[\frac{1}{2} \ln \pi - \frac{1}{2} \psi(D_t/2) - \frac{C}{D_P} + \frac{C}{D_t} \right] + (D_t - 3) \frac{\dot{a}}{a}. \quad (38)$$

Table 3: Values of D_P , C and $\dot{G}_{(1+3)}/G_{(1+3)}|_0$ in terms of sec^{-1} and yr^{-1} .

| D_P | C | $\dot{G}_{(1+3)}/G_{(1+3)} _0 (\text{sec}^{-1})$ | $\dot{G}_{(1+3)}/G_{(1+3)} _0 (\text{yr}^{-1})$ |
|-----------|-----------|--|---|
| 3 | $+\infty$ | 0 | 0 |
| 4 | 1678.797 | $-2.4403 \times 10^{-18} h_0$ | $-7.7006 \times 10^{-11} h_0$ |
| 10 | 599.571 | $-6.8328 \times 10^{-18} h_0$ | $-2.1562 \times 10^{-10} h_0$ |
| 25 | 476.931 | $-8.5899 \times 10^{-18} h_0$ | $-2.7106 \times 10^{-10} h_0$ |
| $+\infty$ | 419.699 | $-9.7612 \times 10^{-18} h_0$ | $-3.0802 \times 10^{-10} h_0$ |

In the limit of constant 3-space dimension, $C \rightarrow +\infty$, and $D_t = D_P = 3$, Eq. (38) leads to the time variation of Newton's constant today

$$\left. \frac{\dot{G}_{(1+3)}}{G_{(1+3)}} \right|_0 = 0. \quad (39)$$

This means that Newton's constant today must be a constant if the space dimension does not change with time. Using Eq. (38) and $H_0^{-1} = 3000 h_0^{-1} \text{Mpc} = 9.2503 \times 10^{27} h_0^{-1} \text{cm}$ corresponding to the Hubble constant today

$$H_0 = 3.2409 \times 10^{-18} h_0 \text{sec}^{-1} = 1.0227 \times 10^{-10} h_0 \text{yr}^{-1}, \quad (40)$$

TVSD theory predicts a decrease in the present value of $G_{(1+3)}$ during the time and derives the time variation of Newton's constant today when $D_P = 4, 10, 25, +\infty$. As shown in Table 3, the effect of time-variable space dimension is that the absolute value of $\dot{G}_{(1+3)}/G_{(1+3)}$ to be bigger than that of in the constant 3-space dimension. In other words, in constant 3-space dimension Newton's constant changes with time less than in the case of time variable space dimension.

3.3 Observational limits on the time variation of the space dimension

According to Ref. [32], the time variation of the gravitational constant today has an upper limit

$$\left| \frac{\dot{G}_{(1+3)}}{G_{(1+3)}} \right|_0 < 9 \times 10^{-12} \text{yr}^{-1}$$

$$< 3 \times 10^{-19} \text{sec}^{-1}. \quad (41)$$

Using Eq. (37) we get the time variation of the space dimension today

$$\begin{aligned} \dot{D}_t \Big|_0 &> \frac{-9 \times 10^{-12} - (D_0 - 3)H_0}{\frac{1}{2} \ln \pi - \frac{1}{2} \psi(D_0/2) + \ln(a_0/\ell_P)} \text{yr}^{-1} \\ &> \frac{-3 \times 10^{-19} - (D_0 - 3)H_0}{\frac{1}{2} \ln \pi - \frac{1}{2} \psi(D_0/2) + \ln(a_0/\ell_P)} \text{sec}^{-1}. \end{aligned} \quad (42)$$

Taking $D_0 = 3$ we get a lower limit on the time variation of the space dimension today

$$\dot{D}_t \Big|_0 > -6 \times 10^{-14} \text{yr}^{-1} \quad (43)$$

$$> -2 \times 10^{-21} \text{sec}^{-1}. \quad (44)$$

These values corresponds to a lower limit for the C -parameter in the theory $C > 14365$. Substituting this value of C in Eq. (4) and taking $a = \ell_P$ we get for the space dimension at the Planck length to be $D_P \leq 3.09$. This means that according to the limits on the time variation of the $(1+3)$ -dimensional Newton's constant, when the scale factor of the Universe is equal to the Planck length, the space dimension of the Universe must be equal to or less than 3.09.

Let us now consider the data from Cassini spacecraft for the post-Newtonian parameter γ [42]

$$\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}. \quad (45)$$

Cassini's experiment has a too short duration and there is no way one could deduce anything about the cosmological time variation of Newton's constant. Also, the uncertainty in the spacecraft state vector (a few kilometers) is too big to allow a good determination of the spacecraft longitude. Cassini's data [42] had nothing to do with the time variation of Newton's constant. For the time variation of Newton's constant one needs much longer experiments, like in the Lunar Laser Ranging experiment. Furthermore, the value of γ , as given in Eq. (45), can give a constraint on the time variation of Newton's constant, but this constraint will be model-dependent [43]. For instance, if we assume a Brans-Dicke theory then $\gamma = (1 + \omega)/(2 + \omega)$ and Newton's constant will vary as t^{-n} with $n^{-1} = 2 + \frac{3\omega}{2}$. If we use γ from Eq. (45),

we get that $|\omega| > 50000$, so that $|\frac{\dot{G}}{G}|_0 < \frac{n}{t_{\text{universe}}} \sim \frac{1}{\frac{75000}{15 \times 10^9 \text{yr}}} \sim 9 \times 10^{-16} \text{yr}^{-1}$. Indeed this is 10000 times smaller than previous bound on time variation of Newton's constant from Ref. [32], see Eq. (41). To get the time variation of the space dimension today based on Cassini's data, the post-Newtonian parameter γ must be calculated in terms of the parameters of TVSD model. Then one can obtain \dot{G}/G today in the model. This issue, like calculations for Brans-Dicke theory in Will's book [43], needs more mathematical treatments and is beyond the aim of this article.

4 Reasons for Ruling the Model Out

This paper presents a new calculation in the context of an unorthodox model in which the dimensionality of space is allowed to vary in a particular prescribed fashion. The model itself is novel but contains some *ad hoc* assumptions. Aside from the mathematical issues associated with what is meant by spacetimes of non-integer dimensionality, the particular manner in which variation of this dimensionality is prescribed - from Eqs. (3), (4) and (5) - is an *ad hoc* assumption which is justified by appealing to polymer physics. Assumptions which work well for polymers, which themselves can be ascribed various fractional dimensions while they fill an existing space of fixed dimension, do not necessarily translate to the dimensionality of space itself. We can imagine that one could conceive of many other ways in which the dimension of space could be varied dynamically; and hopefully some of the ways would have more direct physical motivations coming from quantum gravity itself, rather than an arbitrary analogy to another part of physics which may or may not be actually relevant.

Nonetheless, one can sometimes make progress with toy models based on *ad hoc* assumptions. For example, Ref. [33] (Arkani-Hamed *et al.*) makes the *ad hoc* assumption that the effective Planck scale should vary with a distance scale, space becoming effectively higher-dimensional at very short distances. The model that we have studied in Refs. [2, 3, 4, 5] makes a rather different, but equally *ad hoc* assumption, namely that there is a particular Planck scale $\ell_P = 1.6160 \times 10^{-33} \text{cm}$ which is absolutely fixed, and the dimension of space is then prescribed to vary according to Eq. (3), varying from some initial dimension D_P , when the scale factor was $\ell_P = 1.6160 \times 10^{-33} \text{cm}$.

Although these assumptions appear to be rather artificial and not based

on any particular natural grounds one might expect in quantum gravity, they do nonetheless provide the basis for a novel toy model, which has been extensively studied in Refs. [2, 3, 4, 5].

It would appear to the author of this paper to be time to ask the question, can such models be ruled out observationally? This paper provides a *prima facie* case for doing so. In particular, the conclusion of this paper is that based on observational bounds on the present-day variation of Newton’s constant, one would have to conclude that $D_P \leq 3.09$. If the dimension of space when the Universe was “at the Planck scale” is constrained to be fractional and very close to 3, then the whole edifice of this model loses credibility.

The original hope in this model was that the Universe might be described by some higher-dimensional unified theory, (e.g., 11-dimensional M -theory etc), and that “decrumpling” would provide a dynamical alternative to compactification in the usual Kaluza-Klein sense. It is important to study such alternatives since we are really lacking any good understanding of the “compactification transition”, if such a process ever occurred. However, if the result of the calculations is that $D_P \leq 3.09$ then there can be no place for this alternative to compactification in the context of a higher-dimensional theory of gravity, and this particular means of varying the dimension of space dynamically would appear to be effectively ruled out.

These results are perhaps disappointing, and one might ask what assumptions might be relaxed or altered. The constraint that $D_0 = 3$ at the present epoch seems unavoidable. As discussed, e.g. in section 4.8 of Barrow and Tipler [40], one runs into numerous problems if one abandons having three dimensions today: waves do not propagate cleanly, orbits are unstable due to loss of inverse square law etc. On the other hand, the assumption that the dimensions change at the constant rate prescribed in Refs.[2, 3, 4, 5], is difficult to justify on physical grounds. One might expect that during the early Universe the dimension rapidly altered and then stabilized. However, one then enters the realm of completely quantum gravity. The aim of our original model was to explore the possibility that the effects of a dynamically changing spatial dimension may still have consequences beyond the very early Universe. This remains possible if we prescribe the rate of change of spacetime dimensionality to be different to that considered here and in Refs.[2, 3, 4, 5]. However, it would be best to have a natural means of prescribing this rate of change, rather than a purely *ad hoc* one.

5 Conclusions

The idea put forward in Ref. [1] that the Universe has a decrumpling expansion and its spatial dimensions decrease with time is a bold one but we believe that daring and speculative ideas like this should be explored.

Recently, the variability of the physical constants for example fine structure constant, Newton's constant and the speed of light have attracted much attention. In this article in the framework of time variable space dimension (TVSD) theory we have studied the time variation of $(1 + 3)$ -dimensional Newton's constant. Our treatments to study this topic is based on the relationship between the $(1 + 3)$ and $(1 + D)$ -dimensional Newton's constants as obtained by Gauss' law and on the "true" Newton's constant in the full higher dimensional theory. Previous studies in Ref. [3] about the effective time variation of $(1 + 3)$ -dimensional Newton's constant in TVSD theory had not been included the effects of the volume of the extra spatial dimensions and the surface area of the unit sphere in D -space dimension.

Our main result is that the absolute value of the change of the spatial dimensions must be less than about 10^{-14}yr^{-1} . This value corresponds to a lower limit for the C -parameter in the theory $C > 14365$. Substituting this value of C in Eq. (4) and taking $a = \ell_P$ we get for the spatial dimension at the Planck length to be $D_P \leq 3.09$. This means that according to the limits on the time variation of the $(1 + 3)$ -dimensional Newton's constant, when the scale factor of the Universe is equal to the Planck length, the space dimension of the Universe must be equal to or less than 3.09.

Based on the time variability of the size of the extra spatial dimensions Barrow in his study [15] has reported the present rate of change of the mean radius of any additional spatial dimensions to be less than about 10^{-19}yr^{-1} . It is worth mentioning that Barrow's study is based on the dynamical behavior of the size of extra spatial dimensions while in TVSD theory we take the size of extra spatial dimensions to be constant and the number of the spatial dimensions changes with time.

Indeed, there are certainly many deep physical issues to be explored in the context of TVSD theory. The question of quantum mechanical generation of perturbation and their subsequent evolution is of utmost importance. One can comment upon these issues also the WMAP results and the spectral tilt in the context of TVSD model. One another important question is that what implications are there for Planck epoch if the spatial dimension at the Planck

length (when the scale factor to be ℓ_P) to be 3.02, or what implications are there for primordial nucleosynthesis $z \simeq 10^{10}$ if the spatial dimension at the nucleosynthesis epoch is about 3.13 as studied in Ref. [39]. Can experimental observations of light element abundances be used to rule out any of these models? Similar questions would apply at all other particular redshift of cosmological significance, e.g. at recombination $z \simeq 1000$ etc. It would also be interesting to study a possible variation of the fine structure constant in the context of TVSD theories.

Our results show, however, that some of the fundamental assumptions of the TVSD model, as developed in Refs.[2, 3, 4, 5], need to be altered before these interesting physical questions could be addressed. While it is common to make *ad hoc* assumptions in cosmological model building in the absence of a complete theory of quantum gravity, some of the particular ingredients which we have assumed owe their physical basis perhaps more to polymer physics than to cosmology. The prescribed rate of change of the spatial dimension, which is crucial to making predictions with the model, is particularly hard to justify physically. It is quite possible that this part of the model should be revised. However, just how this should be done is far from obvious.

In conclusion, we have shown that the TVSD model of Refs.[2, 3, 4, 5], with the constraint that at the present epoch $D_0 = 3$ further constraints the space dimension to be $D \leq 3.09$ at the Planck epoch. This would appear to eliminate the original motivation of the TVSD model, which was to integrate a variable space dimension with M -theory as an alternative to compactification, assuming that the spatial dimension was higher at the Planck epoch. Progress with the TVSD model can only be made if there is a breakthrough in terms of finding a natural mechanism for varying the spatial dimension in some alternative fashion to that which we have considered.

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